

Dimensional Analysis Relationships of Geometry Hydraulic Properties For Meandering River in Al Abbasia Reach in Euphrates River

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Abstract — Most of the hydraulic geometry relationships derived under premises that there are direct or indirect relation, at least statistically, between meander geometry characteristics and some hydraulic variables as discharge and velocity. The authors, based on-a-site investigation on Al-Abbasia reach, in the middle of the Euphrates river, Najaf governorate, developed power functions (four models). The study-reach is about six kilometers, it is divided into twenty one cross-sections. These sections represent the meanders and bends in the reach. The recent work is to develop models depending on dimensional analysis and Buckingham Pi theorem. These models are correlate the river width and mean depth with other geometry and hydraulic characteristics. The statistical comparison of the different methods illustrate that the method of dimensional analysis gives higher results in width model comparing with method of power function and lower in mean depth, but acceptable.

Index Terms— Euphrates, Al Abbasia, Najaf; Dimensional Analysis, River Geometry, River Hydraulics, River Meandering, River flow.

1 INTRODUCTION

THE first step in modeling of any physical phenomena is the identification of the relevant variables, and then relating these variables via known physical laws, and one of the most powerful modeling methods is dimensional analysis. Dimensional analysis is a method for reducing the number and complexity of variables which affect a given physical phenomenon, by using a sort of compacting technique (Frank,1997).[1]

Hydraulic - geometry parameters include width, depth, cross sectional area, and meander length, and other hydraulic variables such as mean slope, friction, and mean velocity which depends on many factors like discharge, and type of bed material (Singh, 2003).[2]

Most of the hydraulic geometry relationships derived under premises that there are direct or indirect relation, at least statistically, between meander geometry characteristics and some hydraulic variables as discharge and velocity. The mathematical and statistical methods that define these relations beginning in growing since (1953). Meandering river and Hydraulic geometry has been one of the most explored and investigated topics in hydraulic engineering. No less than Albert Einstein postulated in 1926 a theory explaining the process of meandering on the basis of simple physical laws. Since then understanding of the process has traversed from simple physics to equilibrium and geomorphic theories on one hand and from empiricism to complex mathematical modelling on the other, and yet without a final word as to-

why and how rivers meander. The real impetus toward for-

mulating a theory of hydraulic geometry was provided by the

work of and Maddock (1953). A number of theories have since been proposed. All theories, however, assume that the river flow is steady and uniform and the river tends to attain a state of equilibrium or quasi-equilibrium. The differences are due to the differences in hydraulic mechanisms that the theories employ to explain the attainment of equilibrium by the river. Generally, all the theories can explain how the meandering rivers continue to meander but fail to explain how meanders initiate. Next one exhibits primal theories in hydraulic geometry, (Singh, 2003).[2]

One of the key steps in the process of mathematical modeling is to determine the relationship between the variables. Considering the dimensions of those quantities can be useful when determining such relationship. Dimensional analysis is a method for helping determine how variables are related and for simplifying a mathematical model. Dimensional analysis alone does not give the exact form of an equation, but it can lead to a significant reduction of the number of variables. It is based on two assumptions:

1. Physical quantities have dimensions (fundamental are mass M, length L, and time T). Any physical quantity has a dimension which is a product of powers of the basic dimensions M, L and T;
2. Physical laws are unaltered when changing the units measuring the dimensions.

Units must be taken into consideration when collecting the data as well as when making the list of factors impacting the model and when testing the model. You must check that all the equations in a model are dimensionally consistent. Nalder, G., 1997 [3]

Buckingham Pi theorem is a procedure for determining dimensionless groups from the variables in the problem. The Buckingham Pi Theorem puts the 'method of dimensions' first proposed by Lord Rayleigh in his book "The Theory of Sound" (1877) on a solid theoretical basis, and is based on

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ideas of matrix algebra and concept of the 'rank' of non-square matrices which you may see in math classes. Although it is credited to E. Buckingham (1914), in fact, White points out that the theorem has also appeared earlier in independent publications by A. Vaschy (1892) and D. Riabouchinsky (1911).[4]

The recent paper focuses on developing a dimensional analysis models for width (W) and mean depth (D_m) to link the different geometric and hydraulic characteristics of the river meanders in the selected river reach.

2 PROBLEM DEFINITION

The authors were developed power function models for the hydraulic geometry in the selected reach (4 models) then were compared with other power functions models in previous studies. This work was published in a paper.[5] Using the dimensional analysis with Buckingham Pi theorem, the authors developed another multi-variables models for predicting the hydraulic geometry in the same selected reach.

3 SELECTION OF THE REACH

Al-Abbasia reach along the middle part of the Euphrates river was selected to investigate the different geometry hydraulic characteristics. This region is approximately (6000 m) located between Latitudes (32.04° - 32.03°) and Longitudes (44.26° - 44.29°). This selected reach was divided into 21 sections to perform the field work which included measurement of the hydraulic characteristics of the river sections and longitudinal slopes of the stream and soil sampling. Plate (1) shows the selected sections.



Plate (1): The Selected Reach and Sections.

4 MODELING

4.1 Dimensional Variables

A dimensional analysis of the problem will provide an evaluation in non-dimensional terms which will be completely general. A study of the conditions of flow reveals the problem to be a consideration of the following variables :

- Variables describing geometry of channels.(X)
 - Width of water surface, W
 - Mean depth of flow in channel, D_m

- Variables of flow properties.
 - Discharge, Q
- Variables of fluid properties.
 - Mass density of water, ρ
 - Kinematic viscosity of water, ν
- Variables of sediment properties.
 - Mean size of bed material, d_{50}
 - Specific Gravity G_s
- Variables of flow production.
 - Acceleration due to gravity, g
 - Main slope of stream, S

4.2 Data Limitations

Table (1) lists the limitations of the different characteristics of the selected reach (Al-Abbasia) in Euphrates river to perform the analysis in order to produce models for width (W) and mean depth (D_m). These characteristics were including discharge (Q), velocity (V), area of cross-sections (A), width of water surface (W), mean depth (D_m), max. depth (D_{max}), Main channel slopes (S), mean size of bed material (d_{50}), specific gravity (G_s), and viscosity (ν).

Table (1):
Limitations of The Characteristics In Al-Abbasia Reach.

No.	Characteristics	Symbols	Limitations	Units
1	Mean Depth	D_m	1.7 - 4.5	m
2	Discharge	Q	34 - 78	m ³ /sec
3	Width of the river	W	48 - 184	m
4	Area of cross-sections	A	135 - 535	m ²
5	Average Flow Velocities	V	0.1 - 0.4	m/sec
6	Median sediment size	d_{50}	0.16 - 0.34	Mm
7	Main channel slopes	S	2×10^{-5} - 0.02	-
8	Maximum Depth	D_{max}	2.5 - 9	m
9	Viscosity	ν	2×10^{-6} - 7×10^{-7}	m ² /sec
10	Temperature	T	5 - 36	C°
11	Specific Gravity	G_s	2.61 - 2.75	-

Many parameters as (Ground acceleration, the density of water and others) were considered fixed within this analysis either because they are Low changes or that the change does not affect the results, therefore them fixed to facilitate the calculations and comparison. finally, steady flow assumed for analysis operations in this research.

4.3 Modeling Procedure

If the geometry of channel parameters are taken to be the dependent variable, and the symbol X has been adopted to signify the entry of any one of dependent variable then :

$$X = \phi(Q, G_s, d_{50}, \rho, \nu, g, S) \quad \dots (1)$$

$$f(X, Q, G_s, d_{50}, \rho, \nu, g, S) = \text{constant} \quad \dots (2)$$

Where:

Q : Discharge of water.

G_s : Specific gravity .

d_{50} : Mean size of particle .

- ρ : Density of water.
 ν : Viscosity of water.
 g : Acceleration due to gravity .
 S : Main slope of river.

The number of primary dimensions involved is (3), i.e., $m=3$ (M, L, T). The number of variable is (8), as in Table (1), i. e., $n=8$ Therefore, the number of Π - terms $8-3=5$, thus:

$$F\{\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5\} = \text{constant} \dots (3)$$

Now taking G_s , g and ρ repeating variables:

$$\Pi_1 = Q^{a_1} \cdot d_{50}^{b_1} \cdot \rho^{c_1} \cdot g \dots (4)$$

$$\Pi_2 = Q^{a_2} \cdot d_{50}^{b_2} \cdot \rho^{c_2} \cdot G_s \dots (5)$$

$$\Pi_3 = Q^{a_3} \cdot d_{50}^{b_3} \cdot \rho^{c_3} \cdot \nu \dots (6)$$

$$\Pi_4 = Q^{a_4} \cdot d_{50}^{b_4} \cdot \rho^{c_4} \cdot S \dots (7)$$

$$\Pi_5 = Q^{a_5} \cdot d_{50}^{b_5} \cdot \rho^{c_5} \cdot X \dots (8)$$

Table (2): Primary Dimensions for Each Variables.

	Q	d_{50}	ρ	g	G_s	ν	S	X
L	3	1	-3	1	0	2	0	1
M	0	0	1	0	0	0	0	0
T	-1	0	0	-2	0	-1	0	0

The exponents can be determined under Writing the dimensions as in below :

$$\Pi_1 = Q^{a_1} \cdot d_{50}^{b_1} \cdot \rho^{c_1} \cdot g \dots (9)$$

$$[M^0 L^3 T^0] = \left[\frac{L^3}{T^{-1}} \right]^{a_1} \left[\frac{L}{1} \right]^{b_1} \left[\frac{M}{L^3} \right]^{c_1} \left[\frac{L}{T^{-2}} \right] \dots (10)$$

Equating exponents of M, L and T

For M: $0=c_1 \rightarrow c_1=0$

$$L: 0=3a_1+b_1-3c_1+1 \rightarrow b_1=-3a_1-1$$

$$T: 0=-1a_1-2 \rightarrow a_1=-2 \rightarrow b_1=5$$

$$\Pi_1 = \frac{g d_{50}^5}{Q^2} \dots (11)$$

$$\Pi_2 = Q^{a_2} \cdot d_{50}^{b_2} \cdot \rho^{c_2} \cdot G_s \dots (12)$$

$$[M^0 L^3 T^0] = \left[\frac{L^3}{T^{-1}} \right]^{a_2} \left[\frac{L}{1} \right]^{b_2} \left[\frac{M}{L^3} \right]^{c_2} [1] \dots (13)$$

For M: $0=c_2 \rightarrow c_2=0$

$$L: 0=3a_2+b_2-3c_2 \rightarrow a_2=-b_2/3$$

$$T: 0=-1a_2 \rightarrow a_2=0 \rightarrow b_2=0$$

$$\Pi_2 = G_s \dots (14)$$

$$\Pi_3 = Q^{a_3} \cdot d_{50}^{b_3} \cdot \rho^{c_3} \cdot \nu \dots (15)$$

$$[M^0 L^3 T^0] = \left[\frac{L^3}{T^{-1}} \right]^{a_3} \left[\frac{L}{1} \right]^{b_3} \left[\frac{M}{L^3} \right]^{c_3} \left[\frac{L^2}{T^{-1}} \right] \dots (16)$$

For M: $0=c_3 \rightarrow c_3=0$

$$L: 0=3a_3+b_3-3c_3+2 \rightarrow b_3=-3a_3-2$$

$$T: 0=-1a_3-1 \rightarrow a_3=-1 \rightarrow b_3=1$$

$$\Pi_3 = \frac{\nu d_{50}}{Q} \dots (17)$$

$$\Pi_4 = Q^{a_4} \cdot d_{50}^{b_4} \cdot \rho^{c_4} \cdot S \dots (18)$$

$$[M^0 L^3 T^0] = \left[\frac{L^3}{T^{-1}} \right]^{a_4} \left[\frac{L}{1} \right]^{b_4} \left[\frac{M}{L^3} \right]^{c_4} [1] \dots (19)$$

For M: $0=c_4 \rightarrow c_4=0$

$$L: 0=3a_4+b_4-3c_4 \rightarrow b_4=-3a_4$$

$$T: 0=-1a_4 \rightarrow a_4=0 \rightarrow b_4=0$$

$$\Pi_4 = S \dots (20)$$

$$\Pi_5 = Q^{a_5} \cdot d_{50}^{b_5} \cdot \rho^{c_5} \cdot X \dots (21)$$

$$[M^0 L^3 T^0] = \left[\frac{L^3}{T^{-1}} \right]^{a_5} \left[\frac{L}{1} \right]^{b_5} \left[\frac{M}{L^3} \right]^{c_5} [L] \dots (22)$$

For M: $0=c_5 \rightarrow c_5=0$

$$L: 0=3a_5+b_5-3c_5+1 \rightarrow b_5=-3a_5-1$$

$$T: 0=-1a_5 \rightarrow a_5=0 \rightarrow b_5=-1$$

$$\Pi_5 = \frac{X}{d_{50}} \dots (23)$$

Table (2) illustrates the summary of the expressions for each Π .

Table (2): Expressions for each Π .

Π	Π_1	Π_2	Π_3	Π_4	Π_5
parameter	$\frac{g d_{50}^5}{Q^2}$	G_s	$\frac{\nu d_{50}}{Q}$	S	$\frac{X}{d_{50}}$

Then relationships can be expressed as following :

$$\frac{X}{d_{50}} = f \left[\left(\frac{g d_{50}^5}{Q^2} \right), (G_s), \left(\frac{\nu d_{50}}{Q} \right), (S) \right] \dots (24)$$

The following procedure was followed to reduce the number of π -terms:

$$\Pi_3 / \Pi_1 = \Pi_6 = \left(\frac{\frac{\nu d_{50}}{Q}}{\frac{g d_{50}^5}{Q^2}} \right) = (\nu Q / g d_{50}^4) \dots (25)$$

Thus the functional relationship becomes:

$$\frac{X}{d_{50}} = f \left[(\nu Q / g d_{50}^4), (G_s), (S) \right] \dots (26)$$

$$\frac{X}{d_{50}} = f \left[(\nu Q / g d_{50}^4), (G_s), (S) \right] \dots (27)$$

The final form of formula has to be determined from the conduct of the nonlinear regression analysis on the observed data.

$$W = 54 \times 10^6 d_{50} (\nu Q / 9.81 d_{50}^4)^{-0.23} (G_s)^{-0.01} (S)^{0.003} \dots (28)$$

$$D_m = 25 \times 10^{-4} d_{50} (\nu Q / 9.81 d_{50}^4)^{0.6} (G_s)^{2.6} (S)^{0.017} \dots (29)$$

5 VERIFICATION OF THE MODELS

The nonlinear regression analysis was conducted and it was found for width of water surface and mean depth by using the following two models, model (28) for reach width with R^2 of 0.93, and model (29) for reach mean depth with R^2 of 0.90.

Figure (1) shows the comparison of the model (28) for reach width (W) corresponding to the observed data. The present multi dependent model illustrates a good correlation of the observed data.

Figure (1): Graphical Comparison of Model (28).

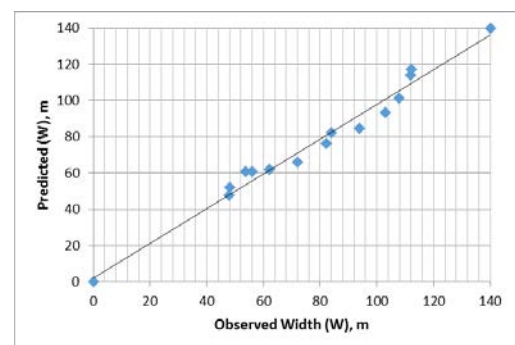
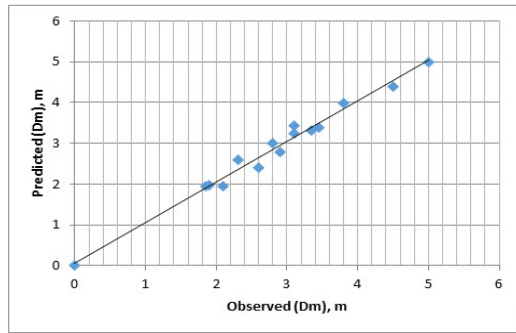


Figure (2) presents the comparison of the model (29) for mean depth of reach (D_m) corresponding to the observed data. The

present multi dependent model demonstrates a good correlation of the observed data.

Figure (2): Graphical Comparison of Model (29).



6 VALIDITY OF THE MODELS

It is obtained depending on data collected from 13 different cross-sections. Other of the 21 cross-sections data are used for verification of produced formula by statistical analysis. A F-test was performed to verify the produced models, model (28), and model (29) using the other eight sections in the selected reach. The results of statistical test revealed that there is a good validity of the models, with F-test 0.90 .

7 COMPARISON OF THE MODELS

F-test was used to evaluate the performance of each models (power function versus dimensional analysis) for width and mean depth through giving the extents of error and acceptance with respect to observed values. F-test (or Fisher distribution) has a minimum of 0, but no maximum value (all values are positive). The largest values indicate better agreement between measured and calculated values.

Table (3) shows the results of F-test for different predicted models. The dimensional analysis gives good F-test results (0.92 for width and 0.75 for mean depth), while the method of dimensional analysis gives higher results in width model comparing with method of power function and lower in mean depth.

Table (3): Values of F-test for Different Models.

Variables	Models	
	Power Function	Dimensional Analysis
Width (W)	0.54	0.92
Mean Depth (D_m)	0.92	0.93

8 CONCLUSION

Dimensional analysis gives two models with good correlation (about 0.9) for both width and mean depth versus the other geometric and hydraulic characteristics. A comparison was made between the models from power function and from dimensional analysis. The dimensional analysis gives good F-test results (0.92 for width and 0.75 for mean depth), while the method of dimensional analysis gives higher results in width model comparing with method of power function and lower

in mean depth.

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